# Correction to Statistical clustering of temporal networks through a dynamic stochastic block model


Catherine Matias and Vincent Miele

Centre National de la Recherche Scientifique, Sorbonne Université, Université Paris Diderot, Laboratoire de Probabilités, Statistique et Modélisation, Paris, France.
catherine.matias@math.cnrs.fr
Université de Lyon, Université Lyon 1, Centre National de la Recherche Scientifique, Laboratoire de Biométrie et Biologie Évolutive, Villeurbanne, France.
vincent.miele@univ-lyon1.fr


This note provides a complement to the proof of the result on parameter identifiability, namely Proposition 1 in the manuscript, in the binary case.

*Complement to the proof of Proposition 1.* We consider the binary setup. The proof from the manuscript establishes that from the distribution $\mathbb{P}_\theta^Y$, we can obtain the parameters $\{\beta_{ql}^t, 1 \leq t \leq T, 1 \leq q, l \leq Q\}$ up to a permutation $\sigma \in \mathfrak{S}_Q$. Note that with the same argument, we also obtain the stationary distribution $\alpha$ on the nodes groups up to the same permutation. Then it remains to identify the parameter $\pi$ of the latent transition matrix. The argument given in the manuscript does not apply to the binary case as it relies on the identifiability of the mixtures of the densities $\phi_{ql}^t$, which are Bernoulli in the binary case (and thus do not induce identifiable mixtures).

Let us consider the degree of the first node at time $t$, namely $D_1^t = \sum_{j=2}^N Y_{1j}^t$ whose conditional distribution, given $Z_1^t = q$ is Binomial $\mathcal{B}(N-1, \bar\beta_q^t)$ where $\bar\beta_q^t = \sum_{l=1}^Q \alpha_l \beta_{ql}^t$. Those parameters $\bar\beta_q^t$ are already identified up to a permutation $\sigma \in \mathfrak{S}_Q$. Now, we consider the distribution of the pair $(D_1^t, D_1^{t+1})$, which is a mixture distribution given by

$$\sum_{1 \leq q,q' \leq Q} \alpha_q \pi(q,q') \mathcal{B}(N-1, \bar\beta_q^t) \otimes \mathcal{B}(N-1, \bar\beta_{q'}^{t+1}).$$

Teicher (1967) has proved the equivalence between parameter identifiability for the mixtures of a family of distributions and parameter identifiability for the mixtures of finite products from this same family. Thus the above mixture has identifiable parameters as soon as the one dimensional mixtures $\sum_{1\leq q \leq Q} \alpha_q \mathcal{B}(N-1, \bar\beta_q^t)$ and $\sum_{1 \leq q' \leq Q}[\sum_{q=1}^Q \alpha_q \pi(q,q')]\mathcal{B}(N-1, \bar\beta_{q'}^{t+1})$ are identifiable. Moreover, Blischke (1964) establishes that these mixtures of Binomial distributions are identifiable as soon as (in our setup) $N-1 \geq 2Q-1$. We thus conclude that we can identify the parameters $\{(\alpha_q\pi(q,q'), \bar\beta_q^t, \bar\beta_{q'}^{t+1}); 1 \leq q, q' \leq Q\}$ up to a permutation on these $Q^2$ values. Nonetheless, the parameters $\alpha_q, \bar\beta_q^t, \bar\beta_{q'}^{t+1}$ are already identified up to a permutation in $\mathfrak{S}_Q$, so we recover the proportions $\{\alpha_q\pi(q,q'); 1 \leq q, q' \leq Q\}$

up to a permutation in $\mathfrak{S}_Q$ and then also the latent transition matrix $\pi$ up to a permutation in $\mathfrak{S}_Q$. $\square$

**Acknowledgments** The authors would like to thank Ann-Kristin Becker for pointing that an argument was missing in the proof.

## References

Blischke, W. (1964). Estimating the parameters of mixtures of binomial distributions. *J. Am. Stat. Assoc. 59*, 510–528.

Teicher, H. (1967). Identifiability of mixtures of product measures. *Ann. Math. Statist 38*, 1300–1302.